What’s Basic in Mathematics?

Thomas C. O’Brien and Ann Moss

Rote memorization of arithmetic “facts” isn’t as important as making sense of math concepts and applying them to the everyday world.

A teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations, he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge and helps them to solve their problems with stimulating questions, he may give them a taste for, and some independent means of, independent thinking.

George Polya, How to Solve It, 1944.

There have been calls for at least 30 years in American education for a return to basics in mathematics instruction. The problem lies in defining what is basic. Math is more than arithmetic. It is a fabric of ideas and relationships and principles and procedures. But all too often the emphasis in classroom instruction is on static facts and procedures. While we have teachers who care deeply about student achievement, too few have spent enough time thinking and learning about and doing math.

IN BRIEF

The authors maintain that mathematics is much more than arithmetic and that teachers should understand and be able to teach mathematical ideas in such a way that children make sense of them and explain, justify, predict, compare, and derive ideas and relate mathematical concepts to the world around them. They provide some game-like activities that challenge participants to solve problems by using some of these concepts.
By 2005 the No Child Left Behind legislation requires schools to have classroom teachers who are “highly qualified” to teach math. A giant step in reaching this goal is to help teachers understand and communicate to children that math is about making sense. If teachers are helped to understand the basic aspects of math, it will be much easier for them to be effective in their instruction.

We can all agree that this is a good idea, but the question remains, “What is it that’s basic in mathematics?” It’s certainly not the rote memorization of arithmetic “facts.” In fact, emphasizing rote memory and rote performance of computational procedures is downright foolish at a time when even a desktop Macintosh can do 12 billion computations in one second—and some mainframes can do 30 trillion per second. Certainly, children should be able to do computation quickly, accurately, and usefully. But mathematics is much more than shopkeeper’s arithmetic.

Making Sense of Mathematics

The most basic thing in mathematics education is to make sense of such concepts as number, distance, angle, quantity, proportion, part-whole, rotation, location, speed, time, size, area, volume, what must be, what might be, and what cannot be. These concepts are all basic to making sense of the world around us by explaining, justifying, predicting, comparing, conjecturing, representing, inventing, classifying, ordering, and inferring.

We know that children can easily learn the thinking patterns that can help them become successful mathematicians in their school world and ultimately in their adult life. But before they can have this opportunity, we must help teachers experience these thinking patterns.

Here are some activities principals can encourage their teachers to try in group problem-solving situations with one another, and then use with children. They challenge the problem-solver to make sense.

Classification
(Preschool and Primary Grades)

- Play “I am thinking of a person,” in which one player thinks of a particular person. The other players are challenged to identify the person not by blurting out “Is it John?” or “Is it Suzie?” but to close in on the identity by asking such questions as “Is the person wearing eyeglasses?” or “Is the person wearing red?” This game teaches young children to think in terms of possibilities rather than specifics.

- Say “I am thinking of a number from 1 to 5. Tell me a number and I’ll tell you whether you’re right, or if you’re too low or too high. You can have as many guesses as you need.” Gradually widen the number range and challenge the players to use as few questions as possible.

Classification and Logical Necessity
(Elementary and Middle Grades)

- Jon writes all the multiples of 6 from 1 to 100. Jan writes all the multiples of 7 from 1 to 100. What is the smallest number to appear on both lists? The biggest number?
- In a certain town there are both painters and poets. Is the oldest painter who is a poet the same person as the oldest poet who is a painter?
- Jane raises both klaxons (three legs) and bloths (five legs). Is it possible for someone to count all the legs and get 44? How many klaxons? How many bloths? Suppose you know there are 12 animals altogether. How does that change things?

Logical necessity is a fancy name for the ability to make a distinction between what must be and what might be. In these activities, the child systematically eliminates possibilities until a single one remains, thus arriving at an answer with logical necessity.

Patterns and Logical Necessity
(Elementary and Middle Grades)

- Two people take turns counting out loud: 1, 2, 3, 4, and so forth. Can you tell, without counting that high, who will get to say 25? How about 36? 200? If that’s too easy, have the counters use multiples of 3, 4, 5, or 6. Or have three people do the counting.

“...Children should be able to do computation quickly, accurately, and usefully. But mathematics is more than shopkeeper’s arithmetic.”
Ms. Jones has a calendar showing all the months in the century. Is it possible to find a month that has exactly three Mondays? Five Mondays? What is the greatest number of Mondays that can occur in a year? What is the smallest number of Mondays that can occur in a year?

The general idea here is to find an underlying principle rather than to rely on a laundry list of information. Try the problems by acting them out, looking for principles and regularities. The mathematician George Polya once said, “Mathematics is being lazy. Mathematics is letting the principles do the work for you so that you don’t have to do it yourself.”

When we challenge children with problems such as these, we cause them to construct or extend or revise their mental networks. And that’s what should be basic in mathematics instruction.

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WEB RESOURCES

ETA Cuisenaire posts samples of Thomas O’Brien’s Daily Tantalizers, math activities that help develop critical thinking skills.

www.etacuisenaire.com/dailytantalizers/dailytantalizers.jsp

Mathematically Sane is an independent site dedicated to the reform of mathematics education.

www.mathematicallysane.com/home.asp

Here are some other Web sites offering high-quality ideas and activities for classroom mathematics:

www.ed.gov/pubs/parents/Math/funmath.html

http://lawrencehallofscience.org/equals/

http://illuminations.nctm.org/


http://gcschool.org/pages/program/Abacus.html

www.cut-the-knot.org

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Manipulatives: A Hands-on Approach to Math

Many of us can remember back to a time in America when all that teachers expected us to do with our hands in the classroom was to fold them. But now that the power and effectiveness of hands-on instruction has been proven in a wide range of subject areas—particularly math—those days are over. Whether using traditional activities, such as counting with beans or coins, or more sophisticated manipulatives (e.g., geo-boards, tangrams, and pattern blocks), hands-on learning helps students to more readily understand concepts and boosts their self-confidence.

Harnessing the power of manipulatives has proven invaluable in the teaching of mathematics. Students are better able to visualize math concepts and gain insights into necessary fundamentals when they use rods, cubes, and other tools. There are many simple hands-on projects that make math fun and educational for children. Stick bundles, flipbooks, flash cards, and puzzles can be effective tools for teaching number facts. A scale for measuring mass and volume can be built using clay, pencils, Styrofoam cups, and string. Tangrams and fraction pieces can be cut from foam or construction paper.

Mixing paint colors brings the concept of ratio alive. For example, one part yellow to two parts blue makes a different green than two parts blue to one part yellow. These results can be charted so other students can get the same results. Children also can make probability games by coloring different shapes on cards and then charting how often a particular shape comes up. Learning becomes interactive and engaging as students become comfortable with their unique learning styles through these active learning experiences.

Principal Thomas Pesce of Stratfield Elementary School in Fairfield, Connecticut, taught math and science before becoming an administrator. He feels that utilizing a multisensory approach targets the strongest learning channels of individual students. “This is beneficial for all children regardless of their abilities,” he says, and cites the value of pizza fractions, inchworms, geometric shapes, tiles, Cuisenaire rods, dice, and other manipulative materials in helping students visualize concepts, construct meaning, and integrate a more tangible understanding of abstract facts.

Principals have a key role to play in helping teachers feel confident about using manipulatives in math and other subject areas. It is important to provide them with encouragement, pedagogical support, and professional development workshops.

A new large-scale study confirms what most principals and teachers have long believed. Active learning experiences using manipulatives appear to function as learning anchors that organize and integrate classroom learning, helping make aspects of what students need to learn more visible than abstract, conceptual instruction.

The study, The Academic Value of Hands-on Craft Projects in Elementary Schools, prepared by an independent educational research organization at the request of the Hobby Industry Association, reports findings from its survey of teachers:

- A substantial majority (90 percent) said that hands-on projects help students understand basic ideas, and 82 percent said that handcrafted projects help their students apply information in new situations. 85 percent of the teachers also agreed that long-term hands-on projects give students a greater depth of understanding than more conventional instructional methods.
- While 46 percent of teachers viewed hands-on projects as an effective learning technique for all students, 54 percent said that this approach is particularly well-suited for students who learn more effectively in non-traditional approaches, particularly visual or kinaesthetic learners, slow readers, and students with limited English-language skills.
- The teachers noted significant differences in learning behaviors when students are involved in hands-on projects. They reported increases in student motivation, willingness to ask questions and volunteer information, enthusiasm, and attention to assigned tasks.

Hands-on educational experiences move students beyond the traditional and passive practices of teaching and learning by incorporating creation, expression, and the presentation of ideas. Spectacular results can be achieved when learning is taken off the chalkboard and literally put into the hands of the learners themselves.

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FOR MORE INFORMATION
For the full report of The Academic Value of Hands-on Craft Projects in Elementary Schools, go to www.teacherplace.org.