Making students mathematically proficient requires major shifts in the thinking and training of principals and teachers.

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Faced with the achievement demands of the No Child Left Behind Act and high-stakes testing, principals are being called on to provide leadership that ensures that their students demonstrate mathematical proficiency. To accomplish this goal, it is important for principals to understand what mathematical proficiency is and how they can promote it in their schools.

**In Brief**

The author defines mathematical proficiency as a balance of five components: conceptual understanding; procedural fluency; strategic competence; adaptive reasoning; and productive disposition. She uses five critical questions to address ways that principals can promote proficiency by ensuring that teachers understand mathematics content; identify and teach essential concepts; use effective instructional methods; monitor student learning; and provide suitable instruction for remediation and enrichment.
Mathematical proficiency has five key components and instructional programs should address all of these components in a coherent and balanced manner, rather than focusing on one to the exclusion of another.

*Conceptual understanding* is comprehension of mathematical concepts, operations, and relations.

*Procedural fluency* is skill in carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately.

*Strategic competence* is the ability to formulate, represent, and solve mathematical problems.

*Adaptive reasoning* is the capacity for logical thought, reflection, explanation, and justification.

*Productive disposition* is an habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy (National Research Council 2001).

What can principals do to promote mathematical proficiency in their schools? Here are five questions they should ask themselves:

1. **Do my teachers know the content of elementary mathematics?**
   
   Liping Ma (1999) calls this knowledge *profound understanding of fundamental mathematics*, or PUFM. Teachers with PUFM understand elementary mathematics with depth and breadth and see mathematical concepts as interconnected knowledge packages. Ma compares the distinction between surface knowledge and profound understanding to that of a taxi driver’s intimate knowledge of a city and a tourist following directions. When a problem arises, such as a street being blocked, the taxi driver who knows the streets and where they connect is able to navigate around the problem more efficiently than the tourist who has only directions and street names. This profound understanding differentiates teachers of elementary mathematics who truly understand the content from those who understand it at a surface level. Teachers with PUFM know, for example, what skills and concepts students need to understand a procedure such as subtraction with regrouping.

   Principals should encourage and support professional opportunities that build on and maintain teachers’ mathematical proficiency. However, providing time for teachers to meet is not enough. Teachers need direction on specific goals to accomplish in their mathematics instruction and resources that can help them move forward. For example, selecting books or articles for discussion can enhance content knowledge and promote conversations about mathematics teaching.

   Principals also should facilitate opportunities for teachers to observe each other’s mathematics teaching. These observations can lead to a more
formal process where teams of teachers plan lessons, observe each other’s teaching, debrief following instruction, and discuss evidence of student learning. These collaborations can focus on how to improve the instruction of specific topics, such as fractions, that pose instructional challenges. Staff development that combines mathematics content knowledge and practical teaching strategies is most effective in improving classroom practices.

2. Have we identified the mathematics that is important for students to know?

Good mathematics programs are aligned with state and national standards and are based on essential understandings and processes. Principals should point out to teachers that state mathematics standards required for each grade level are often different from the content of mathematics textbooks. Teachers need to know that it is okay not to teach certain pages or chapters in the textbook that do not address these essential understandings. What should be taught is the mathematics content that the school has identified as important for students to know.

Textbook companies create products that appeal to a broad audience of schools in many states. The second-grade textbook, for example, will often include much more content than is appropriate for this grade level. Principals can help by scheduling time for teachers, working in grade-level teams, to identify and match state standards with the material selected for instruction from the textbooks.

Encourage teachers to pretest students to determine their prior knowledge of mathematics. Identifying what students know prior to instruction prevents repetition of concepts that students have already mastered and allows for more focus on new content and skills. For example, many children learn addition as the first topic of study every school year, from first through sixth grade. This skill should be targeted for mastery in the early

3. What methods are my teachers using in mathematics instruction?

Students’ understanding of mathematics is intimately tied to how they learn content. Principals should discourage teachers from using the “content inventory” (Schoenfeld 1994) model of teaching mathematics, which gives students the impression that mathematics is a set of disjointed facts. In this model, teachers move from one topic to another without ever making connections that show relationships among mathematical ideas and processes.

Teachers must maximize students’ opportunity to learn—often considered
the most important predictor of student achievement (Berliner & Biddle 1995). Opportunity to learn includes such things as working on mathematical problems, gathering and analyzing data, listening to explanations, reading text, or justifying and defending a position (National Research Council 2001).

Teachers can enhance students’ opportunity to learn by having them create graphic representations, make physical models, generate mental pictures, draw pictures and pictographs, and engage in kinesthetic activity, such as the use of manipulatives. The use of such multiple representations has been shown to be an important factor in students’ abilities to understand mathematical constructs. This is a challenge for teachers who lack the competencies to transform mathematical ideas into nonlinguistic representations (Ball 1990).

One of the most underused forms of such representation is visual images. Students should be encouraged to draw pictures and diagrams of mathematical ideas and situations, and teachers must show them how to read and interpret mathematical ideas that are presented visually. Students should be encouraged to draw pictures and diagrams of mathematical ideas and situations, and teachers must show them how to read and interpret mathematical ideas that are presented visually. For example, the concept of 10 can be expressed using 10 fingers, 10 counters or cubes, a picture of 10 frogs, and saying or writing the word “ten.”

4. What evidence do my teachers have that their students are learning mathematics?

Teachers should be made aware of methods for collecting and documenting evidence of student learning in mathematics. There is truth in that old maxim: “Just because you taught it doesn’t mean I learned it!” Principals can schedule training and work sessions that focus on the following methods of providing evidence of learning:

Document evidence of student learning. Some of the ways to accomplish this include pre- and post-assessments of content knowledge, student math notebooks, checklists of acquired skills, anecdotal records, one-on-one student interviews, and math record folders.

Analyze student data. In the absence of more formal statistical procedures, basic numerical data can be used to show differences in pre- and post-assessments and in growth of skills over time. These data can be converted into graphs, which often show trends that might be missed when looking only at the raw student scores. Teachers should look for patterns and trends and keep records of their observations. Interpreting these documents accurately also provides a good resource for reporting to parents.

Use systematic procedures to interpret class performance. Teacher teams can design common quarterly assessments to document students’ achievement of mathematics benchmarks at each grade level. One method for tracking large class data is to record the performance of individual students on a spreadsheet. Examining student progress over several assessments can show trends in individual and class performance. By systematically examining class performance, teachers can identify frequent error patterns and discover gaps in their instruction.

5. How can we remediate and enrich students’ mathematical learning?

An important way to improve mathematical proficiency is to identify gaps in learning for struggling students, and to select new and interesting mathematical challenges for gifted students. Focusing on patterns of need has the potential to improve instructional practices in a school. Recently I had the privilege of working with a mathematics specialist at a Virginia elementary school, where we examined the results of a third-grade mathematics pretest and identified areas of consistent weakness. We then conducted an item analysis of questions students missed and found that a common characteristic of several missed questions was that they contained pictorial images typical of those presented on standardized mathematics tests. While the third-grade teachers used manipulatives and number symbols in their mathematics instruction, they rarely used pictorial images or discussed how to interpret them. The solution was for the teachers to focus on using nonlinguistic representations in their instruction for 10 weeks. At the end of that period, the entire third grade made a statistically significant improvement on the post-test assessment.

Principals can respond to these five questions and promote mathematical proficiency in their schools by:

- Ensuring that their teachers know elementary mathematics content;
Identifying essential mathematics content that students should know;
- Promoting effective instructional methods;
- Providing evidence of student learning; and
- Designing plans to remediate and enrich students’ mathematical proficiency.

These measures are not easily implemented. Each requires major shifts in the thinking of principals and teachers, as well as considerable time and resources. But schools that are able to respond effectively to each question are well on their way to achieving the goal of mathematical proficiency for their students.

**References**


**WEB RESOURCES**

Math.com, an online math resource, provides information on topics ranging from everyday math to calculus and trigonometry. [www.math.com/](http://www.math.com/)

The Eisenhower National Clearinghouse for Mathematics and Science Education (ENC) supports an online information center for K–12 math and science teachers, with Web links to other online resources. [www.enc.org/](http://www.enc.org/)

Math Forum, an online service of Drexel University, supplies information on current research and innovations in math education. [http://mathforum.org/](http://mathforum.org/)

The Mathematics Education Center at George Mason University focuses on applications of concrete and virtual manipulatives and other forms of representation in math instruction. [http://gse.gmu.edu/centers/cscvm/main.html](http://gse.gmu.edu/centers/cscvm/main.html)
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