WHAT PRINCIPALS NEED TO KNOW ABOUT Teaching and Learning Mathematics

“These three outstanding mathematics teachers and renowned leaders—Tim Kanold, Diane Briars, and Skip Fennell—provide specific information for principals who face the challenges of implementing the Common Core State Standards (CCSS). In this comprehensive book, principals will have the resources and perspectives to develop an intentional plan to support their teachers and students in the newly organized content, mathematical practices, instruction, and assessment.”

—Shirley M. Frye, Retired Mathematics Educator, Former President of NCTM and NCSM

“Kanold, Briars, and Fennell, three of the most highly respected mathematics educators in the country, have put together a remarkable book summarizing what we know about mathematics curriculum, instruction, and assessment and examined all three through the lens of the CCSS for mathematics. Anyone in the position of working to improve mathematics education will find this book an invaluable resource.”

—Matt Larson, K–12 Curriculum Specialist for Mathematics, Lincoln Public Schools; Board of Directors, National Council of Teachers of Mathematics

Understanding the common expectations for mathematics teaching and learning is essential for all principals and administrators. What Principals Need to Know About Teaching and Learning Mathematics uncovers the components of a high-quality mathematics program in regard to content, instruction, assessment, and response to intervention. With this book, you’ll learn how to implement the vision of the Common Core State Standards and ensure a challenging and enjoyable mathematics experience for all K–8 learners.

Authors Timothy D. Kanold, Diane J. Briars, and Francis (Skip) Fennell:

• Offer needs assessments for readers to self-appraise their instructional expectations and challenges
• Outline the qualities of effective teacher teams through ongoing professional development
• Recommend ways to involve families in their children’s mathematics learning

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WHAT PRINCIPALS NEED TO KNOW ABOUT

Teaching and Learning Mathematics

TIMOTHY D. KANOLD
DIANE J. BRIARS
FRANCIS (SKIP) FENNELL
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Dr. Fennell is widely published in professional journals and textbooks related to elementary and middle grade mathematics education and has also authored chapters in yearbooks and resource books. In addition, he has played key leadership roles with NCTM, the Research Council for Mathematics Learning, the Mathematical Sciences Education Board, the National Science Foundation, the Maryland Mathematics Commission, the United States National Commission for Mathematics Instruction, and the Association for Mathematics Teacher Educators. He earned his bachelor’s degree from Lock Haven University of Pennsylvania and his PhD from The Pennsylvania State University.

To book Tim, Diane, or Skip for professional development, contact pd@solution-tree.com.
Knowing mathematics is important! Why is it that this subject sends a “nerd-alert” signal even at the elementary school level? Those who actually wore pocket protectors and truly value mathematics find this inexcusable, particularly at a time when we want—make that need—a much greater proportion of our citizenry to be prepared for careers in the fields of science, technology, engineering, and mathematics. Elementary and middle school is the time to engage students in doing mathematics and also to plant the seed of mathematics importance. Young children begin their schooling seemingly loving math; they enjoy number-related adventures and like to play with shapes. They also see connections to mathematics learning as they engage in the myriad of organized activities that appear to invade and extend their school day. Students see and hear connections like “let’s divide into three teams, angle the ball, hit the cutoff man, pass along the diagonal, find the slugging percentage,” and on and on.

Mathematics can, and frankly must, make a difference in the lives of children at the elementary and middle grade levels, and we—all of us—must take advantage of this opportunity. As you move toward implementation of the Common Core State Standards (CCSS), engaging the Mathematical Practices that the CCSS define will help guide the mathematics content domain experiences that form the foundation so critical for further mathematics learning.

As an elementary school principal, assistant principal, or school-based mathematics instructional leader, your mathematics teaching and learning responsibilities can be difficult challenges. In particular, leading mathematics teaching and learning involves understanding and following policy issues unique to the content area. It is undeniable that the public school system is a social institution influenced by policy at the district, state, and federal levels, and we must acknowledge and respect how the social context of the school impacts mathematics teaching and learning.
Policy Expectations

The CCSS represent, for the first time in U.S. history, a set of common expectations for grades K–12. The CCSS have the potential to significantly affect and shift the day-to-day teaching paradigms of elementary school teachers.

The Common Core State Standards for mathematics define what students should understand and be able to do in mathematics in order to be college and career ready by the end of grade 12. Created by a writing team that included mathematicians, mathematics educators, teachers, and mathematics leaders, they provide expectations that strive to develop understanding of and proficiency in important mathematics topics (Council of Chief State School Officers, 2010a). This historic initiative was driven by the National Governors Association and the Council of Chief State School Officers (CCSSO), which maintains the Common Core State Standards website (www.corestandards.org). At the time of this publication, forty-five states have adopted the CCSS and are moving forward in their transitions toward classroom implementation, with the goal that by the 2014–2015 school year, the United States will have a common set of mathematics standards for all students. The potential of this initiative is both challenging and historic.

As we move toward a deeper discussion of mathematics curriculum, instruction, and assessment, we will continuously circle back to these standards.

The appraisal tool in table I.1 provides a way for you to examine the CCSS mathematical content domains and to consider how they relate to your school’s readiness, ranging from the teachers’ content backgrounds to their understanding of the developmental learning issues related to each of the content domains, as well as to domain-based instructional needs regarding planning, teaching, and assessment. For instance, given the importance of fractions in grades 3–5, it would make sense to consider a professional development focus on fractions developmentally, beginning at the first-grade level with expectations such as:

Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. (CCSSO, 2010a, p. 16)

While understanding fractions is an important component of mathematics learning at the middle grade levels, the depth of understanding must occur at the elementary level. Do your grade-level teachers understand the critical importance of this grade 1 expectation? Is the content background of your teachers sufficient to expand the standard for advanced students and consider the prerequisites for students who may not be ready for this particular slice of mathematics at the first-grade level?

While the fraction analysis example is standard specific, domain-by-domain analysis using table I.1 is a likely first step in a professional development session. Teachers could suggest their own readiness levels using yes or no, or the school’s administrative and mathematics teacher
Table 1.1: CCSS Needs Assessment for Mathematics Content, Learning, and Instructional Needs

<table>
<thead>
<tr>
<th>CCSS Mathematics Domains</th>
<th>Content Grade Level(s)</th>
<th>Content Needs (yes/no)</th>
<th>Developmental Learning Needs (yes/no)</th>
<th>Instructional Needs: Planning (yes/no)</th>
<th>Instructional Needs: Teaching (yes/no)</th>
<th>Instructional Needs: Assessment (yes/no)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting and Cardinality</td>
<td>K (only)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operations and Algebraic Thinking</td>
<td>K–5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number and Operations—Base Ten</td>
<td>K–5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number and Operations—Fractions</td>
<td>3–5 (only)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement and Data</td>
<td>K–5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometry</td>
<td>K–8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratios and Proportional Relationships</td>
<td>6, 7 (only)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Number System</td>
<td>6–8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expressions and Equations</td>
<td>6–8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Functions</td>
<td>8 (only)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>6–8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Visit [go.solution-tree.com/leadership](http://go.solution-tree.com/leadership) for a reproducible version of this table.
leader could use the table to define the school’s needs related to particular areas of content emphasis within the CCSS. The table provides a means to consider your current reality. It is certainly a needs-assessment tool but can also be used to monitor implementation and to update your school’s progress on the path toward full CCSS implementation.

The transition to implementation of the CCSS will include use of one of two consortia assessments. At this writing, twenty-four states have indicated an interest in implementing the Partnership for Assessment of Readiness for College and Careers (PARCC) Assessments, and thirty states have signed on to consider use of the SMARTER Balanced Assessments. Note that the total of states expressing interest in the PARCC and SMARTER balanced assessments is greater than the number of states agreeing to implement the Common Core State Standards. This total represents several states that, at present, are considering both assessment plans, realizing that they will need to determine which assessment they will actually use by 2014. The development of both the PARCC and SMARTER balanced assessments is supported by funding from the U.S. Department of Education. Initial assessments are planned for the 2013–2014 school year. Full implementation of these No Child Left Behind replacement assessments for grades 3–8 and high school will occur during the 2014–2015 school year.

The seemingly simultaneous transition and implementation of the CCSS followed by either the PARCC or SMARTER balanced assessments is a significant policy-related challenge for principals. Specific issues will include how this curriculum and related assessment changes will affect teacher observation and evaluation procedures. What can be done to strengthen a school’s mathematics academic profile while also ensuring a challenging mathematics learning experience for an increasingly diverse student population? While the social forces of educational policy may impact daily challenges and work, they also serve to emphasize that the changes you consider must be grounded in addressing the importance of effective mathematics content, learning, instruction, and assessment.

The issues of equity and access must also become important considerations for mathematics education. To ensure a challenging mathematics experience for every learner, every day, principals must find ways to support and encourage improved mathematics achievement and to develop positive adult and student dispositions at each grade level.

**Teacher Teams**

As principal, one of the most effective moves in the transition toward implementation of the CCSS is to require teachers to work on mathematics instruction in collaborative grade-level and cross-level teams. Such teams are discussed throughout this text. Three tasks are necessary to institute grade-level and cross-level collaborative planning: (1) create a schedule that includes regular grade-level and cross-level planning time; (2) set explicit expectations about how to use this planning time, as well as what products should result; and (3) monitor the collaborative work.
Teams should focus on content domain planning. Specifically, for each domain, every grade-level or cross-level team should do the following:

- Understand and agree on major learning goals of a particular topic (such as place value) and the expected degree of student proficiency on the CCSS learning targets and standards.
- Develop common end-of-topic emphasis assessments.
- Analyze or select high-level tasks to engage core concepts or skills for the mathematical topic. (Ideally, such tasks are already in the team’s instructional materials. If not, teachers will need to find or develop such tasks collaboratively, which is much more efficient than doing this in isolation.)
- Create detailed lesson plans to support the implementation of high-level tasks using the CCSS Mathematical Practices (detailed further in appendix A on page 107).
- Debrief implementation of the high-level task lessons. What did the students do? What unexpected responses came up? What went well? What challenges did the students confront?
- Analyze results of the topic’s assessment by discussing how well students achieved the topic goals overall, what instructional modifications are needed for next year, and which individual students need additional support on specific concepts or skills now.

The principal must establish a monitoring and accountability mechanism regarding these expectations. It is important to establish a feedback mechanism to monitor the progress of the grade-level teams. If you have a mathematics specialist or instructional leader, this individual could meet regularly with the grade-level and cross-level teams, as appropriate.

**Chapter Summaries**

The first four chapters each focus on one aspect of a high-quality mathematics program: content, instruction, assessment, and response to intervention. Each of these chapters includes a section on the research informing that topic. The last four chapters look more closely at the overarching concepts of evaluation, professional development, family engagement, and taking action. All chapters include a section on priorities related to that topic.

Chapter 1, “High-Quality Mathematics Content,” answers the question, What mathematical knowledge will teachers need relative to the proposed curriculum changes within the CCSS? This chapter also discusses learning trajectories as related to CCSS progressions and introduces the CCSS Mathematical Practices. As states and school districts transition to the vision of the CCSS, you will need to prioritize needs concerning the content knowledge of your staff. Should you focus on professional development related to fractions first? Would it make more sense to work on place value across all grades? These kinds of questions will push the prioritization process.
Chapter 2, “High-Quality Mathematics Instruction,” addresses the need to identify and analyze concerns specifically related to instruction. You must consider how staff will access and use a variety of instructional tools that will help students develop critical mathematics concepts and skills as well as develop the “habits of mind” and dispositions—the CCSS Mathematical Practices, per se—that will enable them to use mathematics effectively. This chapter also discusses the importance of eight specific instructional strategies that will promote teacher action in the expectations and the vision of the CCSS Mathematical Practices—especially concerning the role of student formative feedback and how to promote students’ positive self-beliefs.

Chapter 3, “High-Quality Mathematics Assessment,” uses a five-step assessment cycle to delineate how staff can collaborate to build proficiency using a variety of formative and summative mathematics assessments. The highly effective principal understands that assessment is no longer defined by and limited to the summative function of unit assessments.

Chapter 4, “High-Quality Mathematics Response to Intervention,” will help guide decisions about developing and implementing intervention programs in classroom and Tier 3-type pull-out programs. It specifically discusses the role of response to intervention (RTI) in mathematics. Teachers who understand how children learn are able to establish environments that support individual and collaborative learning and to actively engage their students in the learning of mathematics.

Chapter 5, “Monitoring, Evaluating, and Improving Instruction,” discusses how to use the observation-conferring-reflection cycle to improve each teacher’s mathematics instruction and to ensure equity of learning opportunities for all students. This chapter also addresses how to use elementary mathematics specialists, academic coaches, or teacher leaders to promote schoolwide improvements in mathematics instruction.

Chapter 6, “Designing Effective Professional Development,” details effective design and use of collaborative teacher teams as part of ongoing professional development. It is necessary for principals to determine how they and, if available, a school-based mathematics specialist, instructional leader, or academic coach can engage all faculty members and other critical staff in improved instructional practice. Like students in the classrooms, teachers feel engaged and supported when they are free to discuss concerns, admit challenges, and try something different.

Chapter 7, “Working With Families,” examines the diversity of families and how it affects mathematics teaching and learning. This chapter also recommends different ways you can engage and involve families in the mathematics program, such as math nights, parent-teacher conferences, and math newsletters, and highlights resources to support parents and caregivers working with their children outside of school.

Chapter 8, “Turning the Mathematics Vision Into Action,” presents guiding questions for developing a highly successful mathematics program in your school. It also discusses the importance of setting priorities, distributing ownership, and celebrating.
This book asks principals to frame the mathematics needs and challenges faced at the school and district levels around content, learning, and instructional expectations. The following chapters take an in-depth look at the principal’s role regarding essential elements in mathematics curriculum, learning, instruction, and assessment. The discussion includes practice- and research-informed insights into the important issues affecting and shaping the field of mathematics education. Visit go.solution-tree.com/leadership for chapter-by-chapter “Digging Deeper” lists of more resources.
As schools and districts transition to the vision of the Common Core State Standards (CCSS), principals must consider what content issues to address in the mathematics curriculum. Looking at high-quality mathematics, they must also understand which essential elements and expectations are taught in their school, as well as how they know they’re being taught.

This chapter focuses on issues involving mathematics content and, to a lesser extent, instructional practices as part of the overall content development. It begins with one of the most important content paradigm shifts of CCSS expectations: drilling deeper into content development and student understanding. At every grade level, faculty will teach fewer mathematics content standards. However, teachers will need to “dig deeper” into each standard as they teach for conceptual understanding and skill proficiency.

**Research That Informs Content**

As you consider the mathematical content needs and priorities at the school level, there are a number of resources that provide the research and best practices to guide and validate decisions related to the content priorities for your students, the development progressions of particular content domains, and the mathematical knowledge your teachers will need to ease your school’s implementation of the CCSS.

As a first step, consider a review of the mathematics content, learning, and instructional recommendations from the *Principles and Standards for School Mathematics* (NCTM, 2000). Existing state mathematics standards were either developed or revised based on the *Principles and Standards for School Mathematics*. The more recent Curriculum Focal Points (NCTM, 2006) were intended for states, school districts, and local schools to begin a discussion around the focus topics in grades K–8. The Curriculum Focal Points provide excellent content insight, particularly since they are, in essence, the critical topics that introduce each grade level’s content discussion within the CCSS.
An important research-based resource to help guide your transition and implementation of the CCSS is the National Research Council's *Adding It Up* (NRC, 2001). This well-respected resource examines the research related to K–8 mathematics teaching and learning and is a reliable source of authority for any school-based mathematics instructional leader.

As you consider the mathematical content expertise and needs of your faculty and staff, the Conference Board of the Mathematical Sciences’ *Mathematical Education of Teachers* (CBMS, 2001) will help you understand the mathematical content background of teachers at all levels and will be a useful guide as you consider teacher background and their grade-level or cross-grade teaching assignments. A 2012 edition of this work is in press.

Implementation of the CCSS will require you and your teachers to carefully consider the pace and depth of particular content domains and standards within the domains, particularly those standards that require student understanding or expect the use of a variety of representations, from manipulative materials to drawings to technological tools. *How People Learn: Brain, Mind, Experience and School* (NRC, 1999), *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (Cross, Woods, & Schweingruber, 2009), the work of Clements, Sarama, Spitler, Lange, and Wolfe (2011), and others will help you when considering the developmental trajectory of the mathematics concepts and skills that promote understanding and lead to fluency at particular grade levels.

Visit go.solution-tree.com/leadership for further resources regarding mathematics content.

**Less Is More**

Mathematics content helps define what teachers teach and what students learn. The CCSS organizes mathematical content according to content domains (CCSSO, 2010a), and as mentioned previously (see table I.1, page 3), one of the talking points of the Common Core State Standards is the need to focus on fewer expectations and standards per grade level.

While this “less is more” story is a good one, the reality is that teaching for conceptual understanding using a variety of instructional tools is no longer something teachers should simply consider—rather, such teaching is now a daily expectation. To illustrate, consider the following fourth-grade standard excerpt from the CCSS content domain Measurement and Data with particular attention to angles and angle measures.

**Geometric Measurement: Understand Concepts of Angle and Measure Angles**

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that
turns through $1/360$ of a circle is called a “one-degree angle,” and can be used to measure angles.

b. An angle that turns through $n$ one-degree angles is said to have an angle measure of $n$ degrees.

6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. (CCSSO, 2010a, pp. 31–32)

This CCSS standard is deeper than many typical fourth-grade student encounters with angles, which tend to ask students to name types of angles—a low-level cognitive task. Also note that actual instructional tools and strategies are suggested as students acquire this level of understanding.

Table 1.1 shows the CCSS content domains for the K–6 mathematics curriculum. Note that the domains Operations and Algebraic Thinking, Number and Operations in Base Ten, Measurement and Data, and Geometry are elements of the elementary school mathematics curricula from kindergarten through grade 5. Also note the special attention given to the domain Counting and Cardinality at the kindergarten level and to Number and Operations—Fractions at grades 3–5.

Table 1.1: Grades K–6 Mathematics Content Domains

<table>
<thead>
<tr>
<th>Kindergarten</th>
<th>Grades 1–2</th>
<th>Grades 3–5</th>
<th>Grade 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting and Cardinality</td>
<td>Operations and Algebraic Thinking</td>
<td>Operations and Algebraic Thinking</td>
<td>Operations and Algebraic Thinking</td>
</tr>
<tr>
<td>Operations and Algebraic Thinking</td>
<td>Number and Operations in Base Ten</td>
<td>Number and Operations in Base Ten</td>
<td>The Number System</td>
</tr>
<tr>
<td>Number and Operations in Base Ten</td>
<td>Number and Operations in Base Ten</td>
<td>Number and Operations—Fractions</td>
<td>Rational and Proportional Relationships</td>
</tr>
<tr>
<td>Measurement and Data</td>
<td>Measurement and Data</td>
<td>Measurement and Data</td>
<td>Statistics and Probability</td>
</tr>
<tr>
<td>Geometry</td>
<td>Geometry</td>
<td>Geometry</td>
<td>Geometry</td>
</tr>
</tbody>
</table>

Visit go.solution-tree.com/leadership for a reproducible version of this table.

Recognize, too, the departure of Operations and Algebraic Thinking, Number and Operations in Base Ten, Number and Operations—Fractions, and Measurement and Data as the mathematics “grows up” at the sixth-grade level. The content domains shift to much more of a pre-algebra focus at this level with attention to the number system, ratio and proportion, and expressions and equations. Similarly, the emphasis on data shifts from the link with
measurement in grades K–5 to a specific Statistics and Probability domain beginning at grade 6. Table 1.1 shows the cross-grade compatibility of the mathematical domains from grades 1–5 and the differences in kindergarten (counting and cardinality), 3–5 (fractions), and 6 (all domains, except geometry). These considerations provide potential beginning content-topic discussions both within and across grade levels.

Table 1.2 identifies the CCSS critical topics of content focus for grades K–6. This is a different layer of mathematics analysis, targeting what’s important regarding content for each grade level. Each critical area within the CCSS draws from the work of the Curriculum Focal Points (NCTM, 2006) and identifies the mathematics that must receive emphasis or focus at particular grades. These critical areas also provide emphasis topics from another perspective—yours. They highlight, for you, the topics of focus or emphasis at particular grade levels. Table 1.2 also allows you to examine your focus across grade levels. Are there topics that are critical at more than one level? Yes! You can use the focal point–related critical topics for administrative walkthroughs, faculty content and pedagogy conversations, professional development needs, and formal observation expectations.

Learning Trajectories and CCSS Progressions

It is critically important for students to understand the mathematics concepts they are learning. Conceptual understanding—understanding the how and why of mathematics—is also a requirement for teachers. Do your teachers know how to teach for understanding of the content? For instance, you should know whether your second-grade teachers are prepared to help students acquire the CCSS second-grade standard 2.NBT.9: “Explain why addition and subtraction strategies work, using place value and the properties of operations” (CCSSO, 2010a, p. 19). As principal, you are responsible for ensuring that such teacher knowledge helps identify whether students have opportunities that engage them in the learning process and instill in them procedural fluency with understanding.

The acquisition of the basic multiplication and division facts has never been unimportant. Fluency is defined as the student’s ability to respond efficiently and accurately to tasks that involve procedures such as recall of addition and related subtraction facts; use of a standard algorithm for addition, subtraction, and multiplication; or division of whole numbers with or without fractions.

Fluency with understanding cannot be assumed, meaning that a student may be efficient and accurate in multiplying 32 × 45, but may not be able to explain why the product is < 2,000. When students truly understand, they will know that ½ × 8 can be solved without a procedure as long as they think about ½ of 8, or 4. The goal when working with operations is fluency with understanding. If students understand that ½ × 8 is just finding half of 8, or 4, without a need to compute other than using mental mathematics, then understanding is connected to fluency.
<table>
<thead>
<tr>
<th>Kindergarten</th>
<th>Grade 1</th>
<th>Grade 2</th>
<th>Grade 3</th>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Representing and comparing whole numbers, initially with sets of objects</td>
<td>1. Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20</td>
<td>1. Extending understanding of base-ten notation</td>
<td>1. Developing understanding of multiplication and division, and strategies for multiplication and division within 100</td>
<td>1. Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication and division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)</td>
<td>1. Connecting ratio and rate to whole number multiplication and division, and using concepts of ratio and rate to solve problems</td>
<td></td>
</tr>
<tr>
<td>2. Describing shapes and space</td>
<td>2. Developing understanding of whole number relationships and place value, including grouping in tens and ones</td>
<td>2. Building fluency with addition and subtraction</td>
<td>2. Developing understanding of fractions, especially unit fractions (fractions with numerator 1)</td>
<td>2. Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers</td>
<td>2. Extending division to two-digit divisors, integrating decimal fractions into the place value system, developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations</td>
<td>2. Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers</td>
</tr>
<tr>
<td>Kindergarten</td>
<td>Grade 1</td>
<td>Grade 2</td>
<td>Grade 3</td>
<td>Grade 4</td>
<td>Grade 5</td>
<td>Grade 6</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------------------------------------------------------------</td>
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<td>--------------------------------------------------------------------------</td>
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</tr>
<tr>
<td></td>
<td>3. Developing understanding of linear measurement and measuring lengths as iterating units</td>
<td>3. Using standard units of measure</td>
<td>3. Developing understanding of the structure of rectangular arrays and of area</td>
<td>3. Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry</td>
<td>3. Developing understanding of volume</td>
<td>3. Writing, interpreting, and using expressions and equations</td>
</tr>
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</tr>
</tbody>
</table>

Visit [go.solution-tree.com/leadership](go.solution-tree.com/leadership) for a reproducible version of this table.
In addition to general research on learning, and specific mathematics learning, there is a rich body of research about how children learn specific mathematics content and the instruction needed to support it. Three of the best sources for summaries of this research include *Adding It Up* (NRC, 2001), the *Second Handbook of Research on Mathematics Teaching and Learning* (Lester, 2007), and *Mathematics Learning in Early Childhood* (Cross et al., 2009).

Researchers have begun to develop learning trajectories (Clements et al., 2011; Confrey, 2008), sometimes called *teaching-learning paths* (Cross et al., 2009), that describe sequences of milestones children typically follow in learning particular mathematics content (such as whole numbers or basic operations). The CCSS used such pathways as the basis for the standards progression—that is, the order in which the standards appear across grades, as well as the standards that should appear together in the same grade.

A core research finding across specific topics is that conceptual understanding facilitates the acquisition of procedural fluency. Thus the CCSS standards progression for the different content domains: (1) develop conceptual understanding, often building on children’s informal knowledge; (2) support conceptual knowledge and develop informal strategies to solve problems within the domain; and (3) refine the informal strategies to develop fluency with standard procedures.

This progression—conceptual understanding to informal strategies to fluency—poses a significant paradigm shift in teaching mathematics for some teachers. In particular, three progressions in the CCSS content domains that differ significantly from current practice are the domains Operations and Algebraic Thinking, Number and Operations in Base Ten, and Number and Operations—Fractions. Faculty will need to be prepared for these changes. In some cases, the changes will be greater emphasis on topics within domains. In other cases, topics that were presented as expectations at a particular grade will be presented earlier. Examples are fractions being more prominent and extended in grades 3–5 and probability not being introduced until grade 7. Another change is the use of representations (for example, base-ten blocks, fraction models, and the number line) to build and extend concepts. The use of such representations is spelled out in the CCSS, and students are expected to use representations (from manipulatives to drawings to technical tools) throughout their quest for understanding key mathematical concepts. These CCSS content domains and the role of varied representations are prime targets for grade-level or school-based professional development.

The content progressions for addition/subtraction and multiplication/division of whole numbers are similar. They begin by providing students opportunities to engage conceptually using informal methods—counting, acting out, drawing pictures, and so forth. They then progress to writing number sentences using the operations symbols and developing more formal strategies for solving problems.

This important sequencing has two major elements:
1. Instruction starts with solving problems and building meaning and understanding of the operations rather than “teaching” the operations out of context first, and then solving word problems as applications.

2. Students work with a wide variety of addition/subtraction and multiplication/division situations, not just ones in which the result is unknown. This will be an important difference for many teachers (see tables 1 and 2, CCSSO, 2010a, pp. 88–89).

The progression to developing computational proficiency with a standard algorithm for the four basic operations is a historic and important outcome of elementary school mathematics. For each operation, students first build conceptual understanding of the operation and the properties and relationships of the operation; students then have several years to develop proficiency with strategies they can justify. Expected proficiency with standard algorithms for addition and subtraction (grade 4), multiplication (grade 5), and division (grade 6) is on a multiyear trajectory to provide students time to develop understandings of why and how these related, but different procedures work.

**CCSS Mathematical Practices**

One of the more important contributions of the 2010 Common Core State Standards is the Mathematical Practices. These rest on the process standards of *Principles and Standards for School Mathematics* (PSSM; NCTM, 2000) and are the strands of mathematical proficiency specified in *Adding It Up* (NRC, 2001). They describe additional content expertise that all teachers must seek as students develop mathematical competence in procedural knowledge and ability to demonstrate understanding. The Mathematical Practices describe ways in which students are to be actively doing mathematics.

Table 1.3 highlights the interrelatedness of the NCTM Process Standards (NCTM, 2000), mathematical proficiency as defined in *Adding It Up* (NRC, 2001), and the Common Core’s Mathematical Practices. Together they suggest how teachers should consider how students are to engage in mathematics learning. Four important proficiency-related considerations expect students to regularly experience mathematics that will nurture and develop reasoning, problem solving, conceptual understanding, and a productive disposition. *Adding It Up* (NRC, 2001) defines *productive disposition* as “having the inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (p. 5). As students grow mathematically, every teacher’s goal should be to nurture a student’s productive disposition.
Table 1.3: Relating the Standards for Mathematical Practice (CCSS) to Strands of Mathematical Proficiency (*Adding It Up*) and the Process Standards (NCTM)

<table>
<thead>
<tr>
<th>Adding It Up—Mathematical Proficiency</th>
<th>CCSS—Mathematical Practices</th>
<th>PSSM—Process Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategic competence</td>
<td>Make sense of problems and persevere in solving them.</td>
<td>Problem solving</td>
</tr>
<tr>
<td>Adaptive reasoning</td>
<td>Reason abstractly and quantitatively.</td>
<td>Reasoning and proof</td>
</tr>
<tr>
<td>Conceptual understanding, procedural fluency, adaptive reasoning</td>
<td>Construct viable arguments and critique the reasoning of others.</td>
<td>Reasoning and proof, communication</td>
</tr>
<tr>
<td>Conceptual understanding, strategic competence</td>
<td>Model with mathematics.</td>
<td>Connections</td>
</tr>
<tr>
<td>Conceptual understanding</td>
<td>Use appropriate tools strategically.</td>
<td>Representation</td>
</tr>
<tr>
<td>Procedural fluency, strategic competence</td>
<td>Attend to precision.</td>
<td>Communication</td>
</tr>
<tr>
<td>Procedural fluency</td>
<td>Look for and make use of structure.</td>
<td>Communication, representation</td>
</tr>
<tr>
<td>Adaptive reasoning</td>
<td>Look for and express regularity in repeated reasoning.</td>
<td>Reasoning and Proof</td>
</tr>
<tr>
<td>Productive disposition</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Table 1.4 (page 18) presents the Mathematical Practices of the Common Core State Standards (see appendix A, page 107) and possible look-for indicators of how the practices may actually occur in the classroom, with possible student and teacher behaviors noted. The look-fors provided in this table are merely examples and can be modified to seek particular student and teacher behaviors across or within the content domains of the CCSS. Such exemplars will help teachers and principals consider the importance of problem solving, reasoning, precision, modeling with mathematics, and instructional tools, as well as the importance of structure, regularity in reasoning, and communicating about the mathematics learned, as exhibited in the classroom. Consider use of the Mathematical Practices as a frame to organize student and teacher expectations when completing informal walkthroughs at your school site. Also consider using a grid similar to table 1.4, but without the suggested classroom indicators. You could then ask your teachers to fill in the blanks. What would they expect their students to do as they engage in the practices? What do they think they should be doing? Are there some practices that will be easier to access than others? Visit [go.solution-tree.com/leadership](http://go.solution-tree.com/leadership) for a blank, reproducible version of this table.

The practices represent the crucial first step in considering the content domains within the CCSS. The Mathematical Practices are thoroughly discussed as part of instruction in chapter 2.
Table 1.4: Mathematical Practices—Look-Fors as Classroom Indicators

<table>
<thead>
<tr>
<th>Mathematical Practices</th>
<th>Look-Fors—Classroom Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>Students: Are engaged in solving problems</td>
</tr>
<tr>
<td></td>
<td>Teacher: Provides time for students to discuss problem solutions</td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively.</td>
<td>Students: Are able to contextualize and/or decontextualize problems</td>
</tr>
<tr>
<td></td>
<td>Teacher: Provides access to and uses appropriate representations (manipulative materials, drawings, online renderings) of problems</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td>Students: Understand and use prior learning in constructing arguments</td>
</tr>
<tr>
<td></td>
<td>Teacher: Provides opportunities for students to listen to or read the conclusions and arguments of others</td>
</tr>
<tr>
<td>4. Model with mathematics.</td>
<td>Students: Analyze and model relationships mathematically (using an expression or equation)</td>
</tr>
<tr>
<td></td>
<td>Teacher: Provides contexts for students to apply the mathematics learned</td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
<td>Students: Have access to and use instructional tools to deepen understanding (manipulative materials, drawing, technological tools)</td>
</tr>
<tr>
<td></td>
<td>Teacher: Uses appropriate tools (manipulatives) instructionally</td>
</tr>
<tr>
<td>6. Attend to precision.</td>
<td>Students: Recognize the need for precision in response to a problem; use appropriate mathematics vocabulary</td>
</tr>
<tr>
<td></td>
<td>Teacher: Emphasizes the importance of precise communication, including appropriate use of mathematical vocabulary; emphasize the importance of accuracy and efficiency in solutions to problems, including use of estimation and mental mathematics, where appropriate</td>
</tr>
<tr>
<td>7. Look for and make use of structure.</td>
<td>Students: Should be encouraged to look for patterns and structure (use of properties; compose and decompose numbers) within mathematics</td>
</tr>
<tr>
<td></td>
<td>Teacher: Provides time for students to discuss patterns that emerge in a problem’s solution</td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
<td>Students: Reason about varied strategies and methods for solving problems; check for the reasonableness of their results</td>
</tr>
<tr>
<td></td>
<td>Teacher: Encourages students to look for and discuss regularity in their reasoning</td>
</tr>
</tbody>
</table>

Source: Fennell, 2011b.

Content Alignment

The principal’s vision for the daily expectations of mathematics content decisions should match the selection and use of instructional tools. In order to determine this alignment, it is necessary for principals to address the following with teacher teams:

- Does your mathematics curriculum embody and emphasize the CCSS student Mathematical Practices? Do the materials you select or use include activities that will engage students in the Mathematical Practices? If so, how?
- Does your school’s mathematics curriculum emphasize content domains and standards that align with the standards and domains of the CCSS?
• Does your mathematics curriculum account for all the mathematical domains and standards of the CCSS in the materials you are selecting or currently using? Are the suggested activities appropriate, particularly as they involve the representation of key concepts and understandings?

• How important is problem solving in mathematics teaching and learning at your school? Do your mathematics curriculum materials align with the NCTM process standard for problem solving and the CCSS Mathematical Practice standard of “make sense of problems and persevere in solving them” and the related reasoning practices?

• Does your mathematics curriculum require your teachers to place an emphasis on conceptual understanding and procedural fluency? Do the curriculum materials you are using provide lessons and additional activities that address the Common Core State Standards—emphasizing conceptual understanding and fluency? How do you know? Is the balance consistent with your expectations?

The following feature box provides an example of what to look for relative to the Mathematical Practice “reason abstractly and quantitatively.” What must occur before you can expect this practice to become part of the student classroom experience? To answer, consider students engaging in the following problem.

### Reasoning Expectations

Cooper had 7 cards of a set of 38 cards. His brother and sister had the rest of the cards, with his brother Cameron having one more card than his sister Mia. How many cards did they each have?

If you think about the CCSS Mathematical Practice “reason abstractly and quantitatively,” consider what you would like students to do here. What kinds of responses would you like to hear regarding their reasoning? How about the following?

a. Well, 7 + some number is 38, so I counted up . . . 8, 9, 10 is 3, then 28 more, so 31 cards for the other two. Hmm. 15 + 16 is 31, so Cameron has 16 and Mia 15.

b. OK, I subtracted 38 – 7 = 31; that’s how much the brother and sister had. Half of 30 is 15 and one more is 16, so 7 + 15 + 16 = 38.

c. I could think 7 + x + x + 1 = 38, since x + 1 is the one more card for Cameron. That would be 2x + 1 + 7 = 38 or 2x + 8 = 38; 2x = 38 – 8; 2x = 30; x = 30/2; x = 15; so 7 + 15 + 16 = 38.

Consider whether each response represents student reasoning and whether some solutions are more abstract than others.

Looking for evidence of reasoning can help you articulate the level of student understanding for these three different student responses, while determining whether they each include abstract and quantitative reasoning. You can then help teachers plan classroom activities and provide problems that engage their students in reasoning and in establishing a classroom climate that promotes and challenges student thinking and accepts a variety of solutions. The issue of importance here is engaging learners, all learners, in mathematical tasks that challenge and promote reasoning.
Priorities for Charting Improvement in Mathematics Content

The charge of the National Mathematics Advisory Panel (NMAP) was to “examine the critical skills and skill progressions for students to acquire competence in algebra” (2008, p. 7). The panel identified Critical Foundations of Algebra, which are prerequisite concepts and skills essentials for students prior to formal coursework in algebra. The Critical Foundations include the following:

1. Fluency with whole numbers
2. Fluency with fractions
3. Particular aspects of geometry and measurement

The NMAP’s Critical Foundations, along with the Curriculum Focal Points (NCTM, 2006), helped frame the content domains of the CCSS. Note that fluency with whole numbers and fractions, and emphasis on particular aspects of geometry and measurement (such as perimeter and area), are bedrock topics and standards within the elementary school curriculum. Your responsibility is to ensure proficiency for these major topic essentials within the domains of the CCSS.

Teachers will need considerable support in aligning the mathematics they teach—their content—with the CCSS. They will also need opportunities to meet in cross-grade teams to discuss the content domain progressions. These meetings should aim to more specifically discuss and plan how teachers will address these progressions, which include experiences ranging from initial exposure to proficiency, across particular grades. The school’s mathematics specialist or instructional leader should assist in supporting a school’s grade-level team’s progress in addressing these content issues.

The CCSS mathematics content, the domains and the related expectations within the CCSS, are the “what” of mathematics instruction; it’s the math. The next chapter addresses the incredibly important “how to” issue—how to effectively design mathematics instruction that fosters student successful learning of the “what.”
WHAT PRINCIPALS NEED TO KNOW ABOUT
Teaching and Learning Mathematics

“These three outstanding mathematics teachers and renowned leaders—Tim Kanold, Diane Briars, and Skip Fennell—provide specific information for principals who face the challenges of implementing the Common Core State Standards (CCSS). In this comprehensive book, principals will have the resources and perspectives to develop an intentional plan to support their teachers and students in the newly organized content, mathematical practices, instruction, and assessment.”

—Shirley M. Frye, Retired Mathematics Educator, Former President of NCTM and NCSM

“This book provides readers with tools and resources to monitor and evaluate teaching to improve mathematics instruction, assessments, response to intervention, and much more in order to raise student achievement . . . A must-have book for all building administrators and teacher evaluators!”

—Mari Muri, Mathematics Consultant, Project to Increase Mastery of Mathematics and Science, Wesleyan University

“Kanold, Briars, and Fennell, three of the most highly respected mathematics educators in the country, have put together a remarkable book summarizing what we know about mathematics curriculum, instruction, and assessment and examined all three through the lens of the CCSS for mathematics . . . Anyone in the position of working to improve mathematics education will find this book an invaluable resource.”

—Matt Larson, K–12 Curriculum Specialist for Mathematics, Lincoln Public Schools; Board of Directors, National Council of Teachers of Mathematics

Understanding the common expectations for mathematics teaching and learning is essential for all principals and administrators. What Principals Need to Know About Teaching and Learning Mathematics uncovers the components of a high-quality mathematics program in regard to content, instruction, assessment, and response to intervention. With this book, you’ll learn how to implement the vision of the Common Core State Standards and ensure a challenging and enjoyable mathematics experience for all K–8 learners.

Authors Timothy D. Kanold, Diane J. Briars, and Francis (Skip) Fennell:
• Offer needs assessments for readers to self-appraise their instructional expectations and challenges
• Outline the qualities of effective teacher teams through ongoing professional development
• Recommend ways to involve families in their children’s mathematics learning

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